5. DANILOV N.N., The principles of optimality in cooperative differential games with non-transferable payoffs. In: Problems of Control and Information Theory, 11, 1, 1982.
6. PETROSYAN L.A. and TOMSKII G.V., The Geometry of Simple Pursuit. Novosibirsk, Nauka, 1983.
7. PETROSYAN L.A., On a set of differential games of survival in space. Dokl. AN SSSR, 161, 1, 1965.

Translated by J.J.D.

PMM U.S.S.R.,Vol.50,No.1,pp.11-16,1986
Printed in Great Britain

0021-8928/86 $10.00+0.00

THE USE OF FIRST INTEGRALS IN PROBLEMS OF SYNTHESIZING OPTIMAL CONTROL SYSTEMS*

V.E. BERBYUK

The problem of synthesizing optimal control of the motion of a non-linear unsteady system is considered. The control quality is evaluated by a functional of mixed type (a Boltz functional) /1/. A method of synthesizing optimal control systems is worked out for systems of variational problems with a fixed time and a free right end, based on the use of first integrals of the equations of a free uncontrolled object. The effectiveness of the proposed method is illustrated by examples. The synthesis problem, i.e. of representing the optimal control as a function of the system coordinates, has been considered in many publications, for instance in /1-9/ etc.

1. Consider a controllable object whose motion is defined by the equations

$$x^{\cdot} = f(x, t) + b(x, t)\, u(x, t) \tag{1.1}$$

where $x = (x_1, \ldots, x_n)$ is an n-dimensional vector of the phase coordinates, a dot denotes differentiation with respect to t; $u = (u_1, \ldots, u_r)$ is an r-dimensional vector of the controlling functions, $f = (f_1, \ldots, f_n)$, $b = (b_{ij})$ $(i = 1, \ldots, n;\ j = 1, \ldots, r)$ are an n-dimensional vector function, and an $n \times r$ functional matrix respectively specified on some open set Ω of Euclidean space E_{n+1}, in which the coordinates of a point are the numbers $x_1, \ldots, x_n, t$. Henceforth we assume that f, b, u are such that function $f_* = f(x, t) + b(x, t)\, u(x, t)$ and its partial derivatives $\partial f_*/\partial x_i$ $(i = 1, 2, \ldots, n)$ exist and are continuous in the open set Ω.

We call the arbitrary function $u(x, t)$ that satisfies the conditions on $f_*(x, t)$ with values in the Euclidean space E_r the admissible control.

Suppose we are given t_1, t_2, the instants of the beginning and end of the control process and let the initial state of the object be

$$x(t_1) = x_a \tag{1.2}$$

We denote by $v_1(x, t), \ldots, v_k(x, t)$, $k \leqslant n$ the independent first integrals /10/ of the equations of motion of the free (uncontrolled) object, i.e. of the system of equations

$$x^{\cdot} = f(x, t) \tag{1.3}$$

Let $W(y_1, \ldots, y_k)$ be a given arbitrary differentiable function. We select as the arguments y_m the first integrals $v_m(x, t)$, and consider the functional

$$\Phi = W\{v[x(t_2), t_2]\} + \frac{1}{2}\int_{t_1}^{t_2} \sum_{j=1}^{r} \left\{k_j \sum_{i=1}^{n} \frac{\partial W[v(x, t)]}{\partial x_i} b_{ij}(x, t)\right\}^2 dt + \frac{1}{2}\int_{t_1}^{t_2} \sum_{j=1}^{r} \left[\frac{u_j(x, t)}{k_j}\right]^2 dt \tag{1.4}$$

where $v(x, t) = \{v_1(x, t), \ldots, v_k(x, t)\}$ is the vector of first integrals and $k_1, \ldots, k_r$ are specified coefficients.

The first term of the functional (1.4) (the terminal part) is a function of the phase coordinates at the end of the control process and of finite instant of time t_2, the second defines the properties of the object itself as well as its control system. The third term of the functional Φ can be interpreted as the costs of controlling the motion of the object /9/.

The physical meaning of the first two terms of the quality criterion (1.4) can be revealed by the specific selection of the function W and the first integrals $v_m(x, t)$. For example, when

*Prikl.Matem.Mekhan.,50,1,17-23,1986

the free object is a conservative mechanical system, and for function $W[v(x,t)]$ the energy integral is selected, the first term of functional (1.4) defines the total mechanical energy of the object at the end of the control process, and the second defines the rate of dissipation of mechanical energy in the controlled motion of the system considered.

Problem 1. To determine the admissible control $u_*(x, t)$ that transfers the object in a given time $t_2 - t_1$ by virtue of (1.1) from the initial state (1.2) to some final state

$$x(t_2) = x_b \tag{1.5}$$

and gives the minimum value to functional (1.4).

In the statement the final state (1.5) is not specified a priori and is determined when solving the problem, i.e. a variational problem is considered with a fixed control time and a free right end.

Problem 1 is solved by the following theorem.

Theorem 1. Let the motion of the object be defined by (1.1) and the initial state by (1.2), and an arbitrary differentiable function $W(y_1, \ldots, y_k)$, $k \leqslant n$ be given. Then the control actions

$$u_{*j} = -k_j^2 \sum_{i=1}^{n} \frac{\partial W[v_1(x,t), \ldots, v_k(x,t)]}{\partial x_i} b_{ij}(x,t), \quad j = 1, \ldots, r \tag{1.6}$$

where $v_m(x, t)$ $(m = 1, \ldots, k)$ are the independent first integrals of the equations of motion of the free object (1.3) provide the absolute minimum to the functional (1.4) which is equal to $W\{v_1[x(t_1), t_1], \ldots, v_k[x(t_1), t_1]\}$.

To prove Theorem 1 we transform functional (1.4) to the form

$$\Phi = W\{v[x(t_2), t_2]\} - \int_{t_1}^{t_2} \sum_{j=1}^{r} u_j(x,t) \sum_{i=1}^{n} \frac{\partial W[v(x,t)]}{\partial x_i} b_{ij}(x,t)\,dt + \tag{1.7}$$

$$\frac{1}{2} \int_{t_1}^{t_2} \sum_{j=1}^{r} \frac{1}{k_j^2} \left\{u_j(x,t) + k_j^2 \sum_{i=1}^{n} \frac{\partial W[v(x,t)]}{\partial x_i} b_{ij}(x,t)\right\}^2 dt$$

Let us calculate the total derivative of the functional $W[v(x,t)]$ with respect to time. By virtue of the equations of motion (1.1), we obtain

$$\frac{dW}{dt} = \sum_{m=1}^{k} \frac{\partial W}{\partial v_m} \left[\frac{\partial v_m}{\partial t} + \sum_{i=1}^{n} \frac{\partial v_m}{\partial x_i}\left(f_i + \sum_{j=1}^{r} b_{ij} u_j\right)\right] \tag{1.8}$$

Functions $v_m(x,t)$ are the first integrals of system (1.3); hence they satisfy the relations

$$\frac{\partial v_m}{\partial t} + \sum_{i=1}^{n} \frac{\partial v_m}{\partial x_i} f_i = 0, \quad m = 1, \ldots, k \tag{1.9}$$

Taking into account (1.9), from (1.8) we obtain

$$\frac{dW}{dt} = \sum_{j=1}^{r} u_j(x,t) \sum_{i=1}^{n} \frac{\partial W[v(x,t)]}{\partial x_i} b_{ij}(x,t) \tag{1.10}$$

We integrate (1.10) with respect to time from t_1 to t_2 and obtain

$$W\{v[x(t_2), t_2]\} = W\{v[x(t_1), t_1]\} + \int_{t_1}^{t_2} \sum_{j=1}^{r} u_j \sum_{i=1}^{n} \frac{\partial W[v(x,t)]}{\partial x_i} b_{ij}\,dt \tag{1.11}$$

Using (1.7) and (1.11) we obtain for functional (1.4) the formula

$$\Phi = W\{v[x(t_1), t_1]\} + \frac{1}{2} \int_{t_1}^{t_2} \sum_{j=1}^{r} \frac{1}{k_j^2} \left\{u_j + k_j^2 \sum_{i=1}^{n} \frac{\partial W[v(x,t)]}{\partial x_i} b_{ij}\right\}^2 dt \tag{1.12}$$

Note that $W\{v[x(t_1), t_1]\}$ is independent of the choice of control $u(x,t)$, since $x(t_1)$ and t_1 are given a priori, and $v(x,t)$ is the vector of the first integrals of the equation of motion of the free object. Taking the above into account, from an analysis of (1.12) it follows that the functional Φ reaches an absolute minimum with controls (1.6), and $\min_u \Phi(u) = \Phi(u_*) = W\{v[x(t_1), t_1]\}$. Theorem 1 is proved and by the same token problem 1 is sovled.

2. Consider an object whose motion is defined by (1.1) with initial conditions (1.2). Let the Boltz functional be specified in the form

$$\Phi = F[x(t_2), t_2] + \frac{1}{2} \int_{t_1}^{t_2} \left\{Q(x,t) + \sum_{j=1}^{r} \left[\frac{u_j(x,t)}{k_j}\right]^2\right\} dt \tag{2.1}$$

where F is a given non-negative function of the phase coordinates at the final instant of time, Q is a given non-negative function of the phase coordinates and the time, t_1, t_2, and k_j are given constants.

Problem 2. To determine the admissible control $u_*(x, t)$ that transfers the object in a given time $t_2 - t_1$ by virtue of (1.1) from the initial state (1.2) to some final state (1.5) and makes the functional (2.1) a minimum.

It is shown in /4, 6/ that a synthesis of the optimal control in problem 2 can be obtained by the Letova-Kalman method, which reduces to solving a non-linear partial differential equation with the boundary condition

$$\frac{\partial V}{\partial t} + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(x, t) - \frac{1}{2} \sum_{j=1}^{r} \left[k_j \sum_{i=1}^{n} b_{ij}(x, t) \frac{\partial V}{\partial x_i} \right]^2 = -\frac{1}{2} Q(x, t) \tag{2.2}$$

$$V[x(t_2), t_2] = F[x(t_2), t_2] \tag{2.3}$$

The integration of (2.2) presents considerable difficulties. For non-linear objects only one general method for its approximate solution is known. It relates to the case of analytic functions f_i, b_{ij} and is the method of power series. These difficulties in solving (2.2) stimulated a search for another version of the method of synthesis - analytic construction using the generalized the general work /6/.

$$\Phi^\circ = \Phi + \frac{1}{2} \int_{t_1}^{t_2} \sum_{j=1}^{r} \left(k_j \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} b_{ij} \right)^2 dt \tag{2.4}$$

where $\Phi(u)$ is defined by formula (2.1) and $V(x, t)$ is the solution of the linear partial differential equation

$$\frac{\partial V}{\partial t} + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(x, t) = -\frac{1}{2} Q(x, t) \tag{2.5}$$

with boundary condition (2.3). A noted in /6/, passing to the criterion of generalized work fundamentally eases solving the problem of obtaining optimal controls (owing to the linearity of the partial differential equation in the function $V(x, t)$). The possibilities and results of applying analytic construction using the criterion of generalized work (2.4) have been considered in detail in /6/ etc.

It should be noted that formulae (1.1), (1.2), (1.5), (2.1), (2.2), and (1.1), (1.2), (1.5), (2.4), (2.5) generally determine various problems of synthesizing optimal control, since the different functionals are determined, including the last term of functional (2.4), only after solving (2.5), i.e. synthesis using the criterion of the generalized work is semi-definite in advance.

Based on the results of Sect.1, a method is proposed below for solving problem 2 using simpler relations, as compared with (2.2) and (2.3) for the synthesizing function.

Theorem 2. Let the motion of the object be defined by (1.1) and initial conditions (1.2) and let there exist the differentiable function $W(y_1, \ldots, y_k)$ which satisfies the relations

$$\sum_{j=1}^{r} \left\{ k_j \sum_{i=1}^{n} \frac{\partial W[v(x, t)]}{\partial x_i} b_{ij}(x, t) \right\}^2 = Q(x, t) \tag{2.6}$$

$$W\{v[x(t_2), t_2]\} = F[x(t_2), t_2] \tag{2.7}$$

where $v(x, t) = \{v_1(x, t), \ldots, v_k(x, t)\}$ is the vector of independent first integrals of the equations of motion of the free object (1.3). Then the equation of the form (1.6) is the solution of problem 2, i.e. it defines the optimal design in the Boltz problem with fixed time and a free right end.

To prove Theorem 2 it is sufficient to express functional (2.1), taking into account (2.6) and (2.7), and to show directly, that the expression obtained is the same as functional (1.4), i.e. that all the conditions of Theorem 1 are satisfied with the consequent validity of Theorem 2.

Thus to determine the optimal control in problem 2 it is necessary to solve for function W the functionally differential Eq.(2.6) with condition (2.7).

Consider a scalar control action, i.e. we set $r = 1$. We assume that the functions $Q(x, t)$, $b_{i1}(x, t)$ admit of the representations

$$k_1 \sum_{i=1}^{n} \frac{\partial v_m(x, t)}{\partial x_i} b_{i1}(x, t) = \varphi_m[v_1(x, t), \ldots, v_k(x, t)]$$

$$\sqrt{Q(x, t)} = \Theta[v_1(x, t), \ldots, v_k(x, t)], \quad m = 1, \ldots, k$$

where φ_m, Θ are some known functions. Then for the design function $W(v_1, \ldots, v_k)$ from (2.6) we have the first-order partial differential equation

$$\sum_{m=1}^{k} \frac{\partial W(v_1, \ldots, v_k)}{\partial v_m} \varphi_m(v_1, \ldots, v_k) = \Theta(v_1, \ldots, v_k) \tag{2.8}$$

Its general solution in implicit form is

$$\Psi[z_1(v_1, \ldots, v_k, W), \ldots, z_k(v_1, \ldots, v_k, W)] = 0 \tag{2.9}$$

where Ψ is an arbitrary differentiable function and $z_1, \ldots, z_k$ are independent first integrals of the system of ordinary differential equations

$$\frac{dv_1}{\varphi_1} = \ldots = \frac{dv_k}{\varphi_k} = \frac{dW}{\Theta}$$

Formula (2.9) enables us to determine the required function W.

In the case of a single first integral of the free-object equations of motion, i.e. $k = 1$, for the synthesizing function $W[v_1(x, t)]$ by virtue of (2.6)–(2.8), we have

$$W = \varphi_*[v_1(x, t)] + F[x(t_2), t_2] - \varphi_*\{v_1[x(t_2), t_2]\},$$
$$\varphi_* = \int \frac{\Theta(v_1)}{\varphi_1(v_1)} dv_1$$

Note that for the existence of the differentiable function $W(y_1, \ldots, y_k)$, satisfying relations (2.6) and (2.7), in the variational problem (1.1), (1.2), (1.5), and (2.1) the phase constraint

$$\sum_{j=1}^{r} \left\{k_j \sum_{i=1}^{n} \frac{\partial F}{\partial x_i(t_2)} b_{ij}[x(t_2), t_2]\right\}^2 = Q[x(t_2), t_2]$$

must necessarily be satisfied.

The above investigations enable the following algorithm of optimal control synthesis in the Boltz problem of the form (1.1), (1.2), (1.5), and (2.1) to be compiled.

1°. Determine the first independent integrals of the free-object equations of motion (1.3) (at least one of the integrals).

2°. Solve the auxiliary boundary value problem (2.6), (2.7) and determine the synthesizing function of the first integrals.

3°. Using formula (1.2) calculate the optimal control, and from (1.1), (1.2) determine the corresponding optimal law of motion of the object.

4°. Calculate the absolute minimum of functional (2.1) using the formula

$$\Phi_{\min} = \Phi(u_*) = W\{v[x(t_1), t_1]\}$$

3. We shall give some examples of the application of the above problem of synthesizing the optimal control of the motion of mechanical systems.

Example 1. Let the motion of the object be defined by the equations

$$x_1^{\cdot} = f_1(x_1, x_2, t, u),\ x_2^{\cdot} = u,\ x(t_1) = x_a \tag{3.1}$$

and the control quality by the functional

$$\Phi = F[x_2(t_2), t_2] + \frac{1}{2} \int_{t_1}^{t_2} [Q(x_2) + u^2(x, t)]\, dt \tag{3.2}$$

where F, Q are given non-negative functions of their arguments, $u(x, t)$ is a scalar controlling action, and t_1, t_2 are fixed instants of the beginning and end of the control process. It is required to determine the admissible control that transfers the object by virtue of (3.1) in a given time $t_2 - t_1$ from the initial state to some final state (1.5) and makes functional (3.2) a minimum.

We solve the problem formulated in accordance with the algorithm proposed in Sect.2.

1°. The independent first integral of the free-body equations of motion $x_1^{\cdot} = f_1(x_1, x_2, t, 0)$, $x_2^{\cdot} = 0$ has the form $v \equiv x_2$.

2°. The auxiliary boundary value problem (2.6), (2.7) reduces in this problem to determining the function $W(x_2)$ that satisfies the equations

$$dW/dx_2 = \sqrt{Q(x_2)},\ W[x_2(t_2)] = F[x_2(t_2), t_2] \tag{3.3}$$

From the solution of (3.3) we have

$$W(x_2) = Q_*(x_2) + F[x_2(t_2), t_2] - Q_*[x_2(t_2)], \qquad Q_*(x_2) = \int \sqrt{Q(x_2)}\, dx_2$$

3°. On the basis of Theorem 2 we conclude that the absolute minimum of functional (3.2) is provided by the control $u_*(x, t) = -dW/dx_2 = -\sqrt{Q(x_2)}$. The corresponding optimum law of motion of the object is determined by the Cauchy problem

$$x_1^{\cdot} = f_1[x_1, x_2, t, -\sqrt{Q(x_2)}],\ x_2^{\cdot} = -\sqrt{Q\ \ (x_2)},\ x(t_1) = x_a$$

The minimum of functional (3.2) is

$$W[x_2(t_1)] = Q_*[x_2(t_1)] + F[x_2(t_2), t_2] - Q_*[x_2(t_2)]$$

Example 2. Consider the motion of a material point of mass m along the Ox_1 axis under the action of a force of potential $\Pi(x_1)$ and the controlling action $u(x_1, x_2, t)$. The equations of motion have the form

$$x_1^{\cdot} = x_2,\ \ mx_2^{\cdot} = -\frac{d\Pi(x_1)}{dx_1} + u(x_1, x_2, t) \tag{3.4}$$

It is required to determine the control $u(x, t)$ that transfers the point by virtue of (3.4) in time $t_2 - t_1$ from the initial state (1.2) to some final state (1.5), and provides the minimum of the functional

$$\Phi = \frac{1}{2} mx_2^2(t_2) + \Pi[x_1(t_2)] + \frac{1}{2}\int_{t_1}^{t_2} x_2^2(t)\,dt + \frac{1}{2}\int_{t_1}^{t_2} u^2(x, t)\,dt \tag{3.5}$$

The terminal part of the functional Φ represents the value of the total mechanical energy of the system at the end of the control process when $t = t_2$, the third term in (3.5) defines the dissipation of mechanical energy during the controlled motion, and the fourth the energy expended on the control.

This problem may be solved using Theorem 1. Indeed the free-body equations of motion (3.4) for $u \equiv 0$ have an independent first integral, i.e. the energy integral $v = {}^1/_2 mx_2^2 + \Pi(x_1)$. If this integral is taken for the design function $W(v)$, and one takes into account that in this problem $b_1 = 0,\ b_2 = 1/m,\ n = 2,\ r = k_1 = 1$, functional (3.5) takes the form (1.4). By Theorem 1 the solution of the problem is

$$u_*(x, t) = -\sum_{i=1}^{2} \frac{\partial v}{\partial x_i} b_i = -x_2(t) \tag{3.6}$$

The functional (3.5) on the control $u_*(x, t)$ reaches an absolute minimum equal to ${}^1/_2 mx_2^2(t_1) + \Pi[x_1(t_1)]$, i.e. the value of total mechanical energy at the instant the control process begins. The corresponding optimal law of motion of the point is determined from the following Cauchy problem:

$$x_1^{\cdot} = x_2,\ mx_2^{\cdot} = -d\Pi(x_1)/dx_1 - x_2,\ x(t_1) = x_a \tag{3.7}$$

For comparison let us solve this problem by the methods of the classical calculus of variations. By the Lagrange principle /11/ it is necessary to follow the following procedure.

1°. Form the Lagrangian L and terminator l. In this problem they have the form

$$L = \frac{\lambda_0}{2}(x_2^2 + u^2) + p_1(x_1^{\cdot} - x_2) + p_2\left(x_2^{\cdot} + \frac{1}{m}\frac{d\Pi}{dx_1} - \frac{u}{m}\right)$$

$$l = \frac{\lambda_0}{2}\{mx_2^2(t_2) + 2\Pi[x_1(t_2)]\} + \lambda_1 x_1(t_1) + \lambda_2 x_2(t_1)$$

where $(\lambda_0, \lambda_1, \lambda_2, p_1, p_2$ are undetermined Lagrange multipliers.

2°. Write down the necessary conditions of optimality of the process (x, u, t_1, t_2):

a) steadiness with respect to x for the Lagrangian L (the Euler equation)

$$p_1^{\cdot} - \frac{p_2}{m}\frac{d^2\Pi}{dx_1^2} = 0,\ \ p_2^{\cdot} - \lambda_0 x_2 + p_1 = 0 \tag{3.8}$$

b) transversality with respect to x for the terminator l:

$$p_1(t_1) = \lambda_1,\ \ p_1(t_2) = -\lambda_0\frac{d\Pi[x_1(t_2)]}{dx_1(t_2)}$$
$$p_2(t_1) = \lambda_2,\ \ p_2(t_2) = -\lambda_0 mx_2(t_2) \tag{3.9}$$

c) steadiness with respect to u of the Lagrangian L

$$\lambda_0 u - p_2/m = 0 \tag{3.10}$$

3°. Determine the admissible controllable processes for which the conditions (3.8)–(3.10) with Lagrangian multipliers λ_j and p_j simultaneously equal zero. Among all the extremal processes obtained find the solution of the problem or prove that there is no solution.

Let us carry out Step 3°. First, let us consider the case when $\lambda_0 = 0$. As follows from (3.8)–(3.10). it is necessary that $\lambda_1 = \lambda_2 = p_1 = p_2 = 0$, i.e. all the Lagrange multipliers are zeros, which means that for $\lambda_0 = 0$ there are no admissible extremals. Assuming $\lambda_0 = 1$ from (3.10) we obtain

$$u = p_2/m \tag{3.11}$$

Taking (3.11) into account, from (3.4) and (3.8) for determining $x_i(t), p_i(t)$ we have

$$x_1^{\cdot} = x_2,\ \ mx_2^{\cdot} = -\frac{d\Pi}{dx_1} + \frac{p_2}{m} \qquad p_1^{\cdot} - \frac{p_2}{m}\frac{d^2\Pi}{dx_1^2} = 0,\ \ p_2^{\cdot} - x_2 + p_1 = 0 \tag{3.12}$$

We select the functions p_i in the form

$$p_1 = -d\Pi/dx_1, \quad p_2 = -mx_2 \tag{3.13}$$

Then, as follows from (3.11)−(3.13), $u_* = -x_2(t)$, and the extremals $x_*(t)$ must be, as before, solutions of the Cauchy problem (3.7). The satisfaction of the transversality conditions (3.9) is guaranteed, if we assume $\lambda_1 = -d\Pi[x_1(t_1)]/dx_1(t_1)$, $\lambda_2 = -mx_2(t_1)$. For the completion of the solution of this problem by the methods of the classical calculus of variations it is further necessary to prove that the control $u_* = -x_2$ obtained provides the minimum of the functional (3.5). This had been proved using Theorem 1.

The examples considered here show the effectiveness of the proposed design of optimal control of the motion of mechanical systems based on the use of the first integrals.

REFERENCES

1. MOISEYEV H.H., Elements of the Theory of Optimal Systems. Moscow, Nauka, 1975.
2. PONTRYAGIN L.S., BOLTYANSKII V.G., GAMKRELIDZE R.V. and MISHCHENKO E.F., The mechanical Theory of Optimal Processes. Moscow, Nauka, 1976.
3. KRASOVSKII N.N., The Theory of Motion Control. Moscow, Nauka, 1968.
4. LETOV A.M., The Dynamics and Control of Flight. Moscow, Nauka, 1969.
5. CHERNOUS'KO F.L. and BANICHUK N.V., Variational Problems of Mechanics and Control. Moscow, Nauka, 1973.
6. KRASOVSKII A.A., Systems of Automatic Flight Control and Their Analytic Construction. Moscow, Nauka, 1973.
7. LARIN V.B., NAUMENKO K.I., and SUNTSEV V.N., The Design of Optimal Linear Systems with Feedback. Kiev. Nauk. Dumka, 1973.
8. KUNTSEVICH V.M. and LYCHAK M.M., Design of Automatic Control Systems Using Lyapunov Functions. Moscow, Nauka, 1977.
9. CHERNOUS'RO F.L., AKULENKO L.D. and SOKOLOV B.N., The Control of Oscillations. Moscow, Nauka, 1980.
10. PONTRYAGIN L.S., Ordinary Differential Equations. Moscow, Nauka, 1974.
11. ALEKSEYEV V.M., TIKHOMIROV V.M. and FOMIN S.V., Optimal Control. Moscow, Nauka, 1979.

Translated by J.J.D.

PMM U.S.S.R.,Vol.50,No.1,pp.16-22,1986
Printed in Great Britain

0021-8928/86 $10.00+0.00

OPTIMAL CONTROL OF STEPWISE PROCESSES WITH PERIODIC CHARACTERISTICS*

A.B. PIUNOVSKII

The problem of optimal control of a stepwise Markov process with periodic characteristics that is not discontinuous with respect to probability is solved. The sufficiency of periodic Markov control strategies is proved, the optimality equation is obtained, and examples of the solution of practical problems are given.

The construction of optimal strategies for the control of stochastic processes is a pressing practical problem. /1—10/. Besides stochastically continuous /1, 2, 5, 7—9/ and purely discontinuous /3, 4, 6/ models of controllable processes, problems in which the controllable stochastic process has a mixed character are of interest. In /1—10/ models with diffusion and intermittent components, and also with other interacting Markov processes were studied. One of the varieties of such combined models, including a chain with discrete time and a stochastically continuous intermittent process are considered in this paper. Problems of the optimal control of such system were investigated in /10/ in a finite time interval. Here the problem of synthesis in an infinite time interval is considered on the assumption that all the characteristics of the controlled model are periodic time functions.

1. Notation and definitions. A two-component Markov intermittent stochastic process (ξ_t, ψ_t) is considered here in an infinite time interval $I = [0, \infty)$. The component ξ_t represents a stochastically continuous process, the jumps of the component ψ_t appear at known instants $\tau, 2\tau, \ldots$. We denote by X the space of component ξ_t, and Y is the space of component states ψ_t that are finite or denumerable sets. The term state of the process (ξ_t, ψ_t) at the instant

Prikl.Matem.Mekhan.,50,1,24-31,1986